Sight reduction following NavPac compact data 2006-2010/HMNAO

1) Calculate dip D_h:

$$D_h = 0^\circ.0293\sqrt{h} = 1'.76\sqrt{h}$$

with: h the height (in m) of the eye above the horizon.

$D_h = 0^\circ.0414 = 2'.48$ (eyeheight 2 m)
$D_h = 0^\circ.0507 = 3'.05$ (eyeheight 3 m)

2) Calculate apparent altitude H:

$$H = H_s + I - D_h$$

with: H_s the altitude read from the sextant
I the sextant index error (determine from horizon-calibration)
D_h the dip as computed under **1)**

3) Calculate the refraction correction R:

$$R = \frac{0^\circ.0162}{\tan H}\left(\times\frac{0.28P}{T+273}\right) = \frac{0'.97}{\tan H}\left(\times\frac{0.28P}{T+273}\right)$$

with: H the apparent altitude as computed under **2)** (in ° !!)
P the pressure (in mbar !!) at time of observation
T the temperature (in centigrade !!) at time of observation

Note: if the pressure and temperature are not known, the factor in brackets can be omitted at the cost of a small fraction of an arcminute (small fraction of Nautical Mile).

4) If using the objects: _Sun, Venus, or Mars_, derive the parallax PA:

$$PA = HP\cos H$$

with: HP the horizontal parallax which amounts for:

Sun: $HP_\odot = 0^\circ.0024 = 0'.14$
Venus ($HP_♀$), Mars ($HP_♂$): Take from table "Horizontal Parallaxes and Semi-Diameters for 2009"

5) If using the _Moon_: derive the parallax PA:

$$PA_☾ = HP_☾\cos H + OB_\oplus$$

with: $OB_\oplus$ the correction for oblateness of the Earth:

$$OB_\oplus = -0°.0017 \cos H = -0'.10 \cos H$$

and:

$HP_☾$: Take from table “GHA and DEC column MOON”

6) If using the *Sun or Moon*: compute semi-diameter S:

Sun ($S_\odot$): Take from table “Horizontal Parallaxes and Semi-Diameters for 2009”

Moon: $S_☾ = 0.2724 \times HP_☾$ with $HP_☾$ as computed under **5)**

7) Compute the observed altitude H_O of the celestial object:

$$H_O = H - R + \underbrace{PA}_{\odot,☾,♀,♂} \pm \underbrace{S}_{\odot,☾}$$

with: H the apparent altitude as computed under **2)**

R the refraction correction as computed under **3)**

PA the parallax as computed under **4)** (***Sun*** $\odot$, ***Venus*** ♀, ***Mars*** ♂), or **5)** (***Moon*** ☾)

S the semi-diameter for the ***Sun*** $\odot$ or the ***Moon*** ☾, where the '$+$' sign should used if the lower limb was observed (in general the case) and the '$-$' sign should be used if the upper limb was observed (when the lower limb could not be used, *e.g.,* due to moon-phase)

Calculation of altitude and azimuth for an estimated position:

1) Estimate the position ($Long$,Lat) on the chart for the time of observation: this doesn't have to be a very good estimate: roughly $\pm 1^\circ$ in latitude and/or longitude suffices, provided the selected celestial objects for position determination have observed altitudes not exceeding $H_O < 70^\circ$. The GP (ground position) of the object is at least 20° (~ 1200 NM) from the actual ship's position and the position line is in good approximation a straight line.

2) Take via linear interpolation the GHA and DEC for the ***Sun***, ***Moon, and Planets*** from the GHA and DEC table for the Month, Day (DD), and Hour (HH).
Formula for linear interpolation GHA(T) at time of observation T = HH:MM:SS is as follows (time in UT !!):

$$GHA(T) = GHA(HH) + \Big(GHA(HH+1) - GHA(HH)\Big) \times \left(\frac{MM}{60} + \frac{SS}{3600}\right)$$

Here GHA(HH+1) and GHA(HH) are the tabulated GHA's at the full hour HH and HH + 1 hour, respectively. MM and SS denote the minutes and seconds at the time of observation that have passed after the full hour HH. Analogously for DEC(T).

For the ***Stars*** compute the GHA from the SHA and GHA of Aries (take interpolation of tabulated numbers for GHA of Aries), either interpolation of tabulated numbers. eclinations for the ***Stars*** are constant.

$$GHA_\star = GHA_{\text{Aries}} + SHA_\star$$

3) Compute the local hour angle LHA:

$$LHA = GHA + Long$$

$Long$ = negative for “W” and is positive for “E”
add/subtract multiples of 360° so that LHA is within 0° – 360°

4) Calculate the computed altitude of the object H_C that should be observed if the ship was at the estimated position:

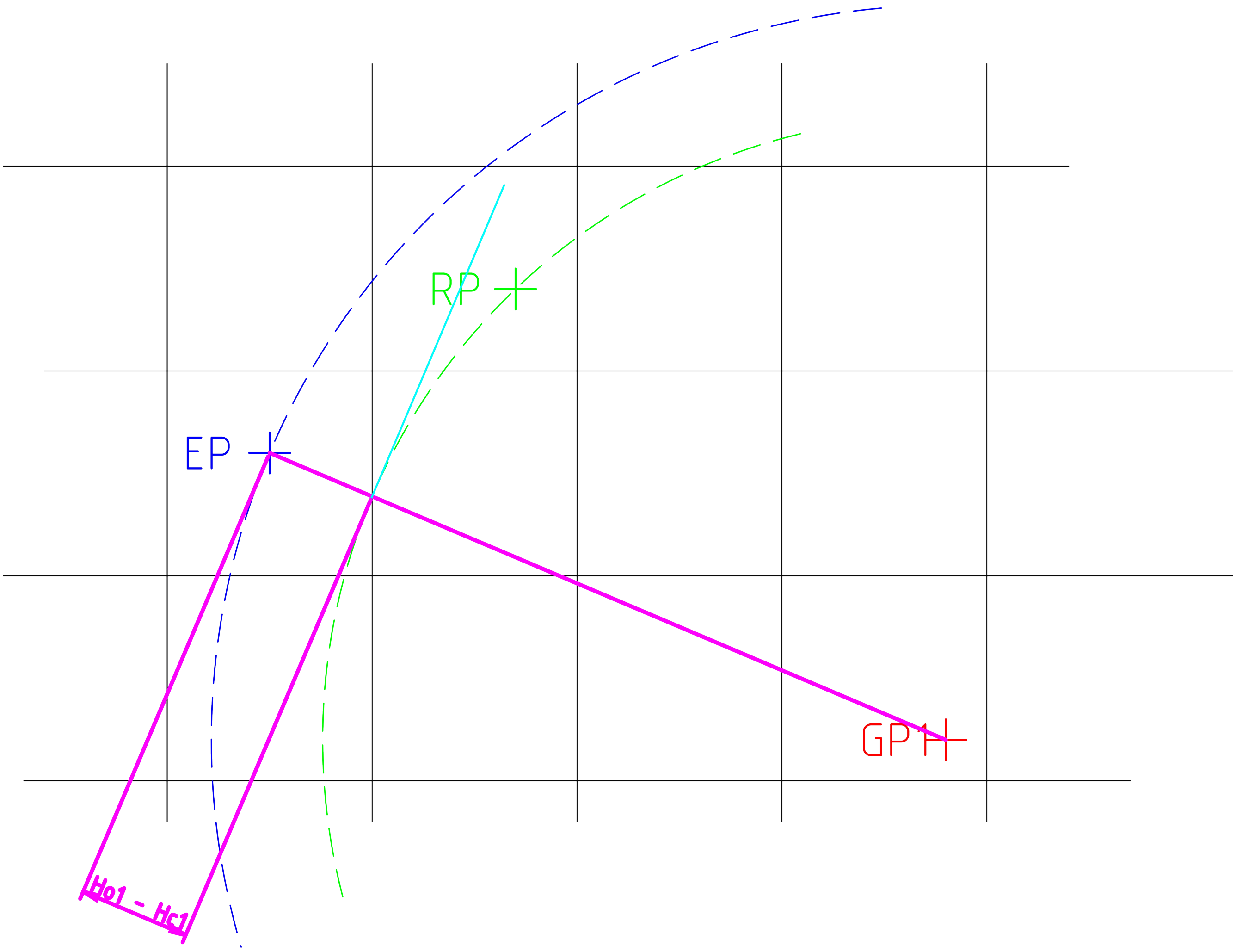
$$H_C = \operatorname{asin}\big(\sin(DEC)\sin(Lat) + \cos(DEC)\cos(Lat)\cos(LHA)\big)$$

5) Compute the azimuth Z:

$$Z = \underbrace{(360^\circ -)}_{\substack{\text{only if} \\ LHA<180^\circ}} \operatorname{acos}\left(\frac{\sin(DEC)\cos(Lat) - \cos(DEC)\sin(Lat)\cos(LHA)}{\cos H_C}\right)$$

if the argument of the arccos is smaller than –1 then set it to –1,
if the argument of the arccos is larger than +1 then set it to +1

Constructing actual position from estimated position, computed altitude, computed azimuth, and observed altitude



In the above picture a piece of a chart with in black: latitude/longitude grid, in blue: estimated position (EP), in green: real position (RP), and in red (ground position of celestial object 1, GP1) have been drawn.

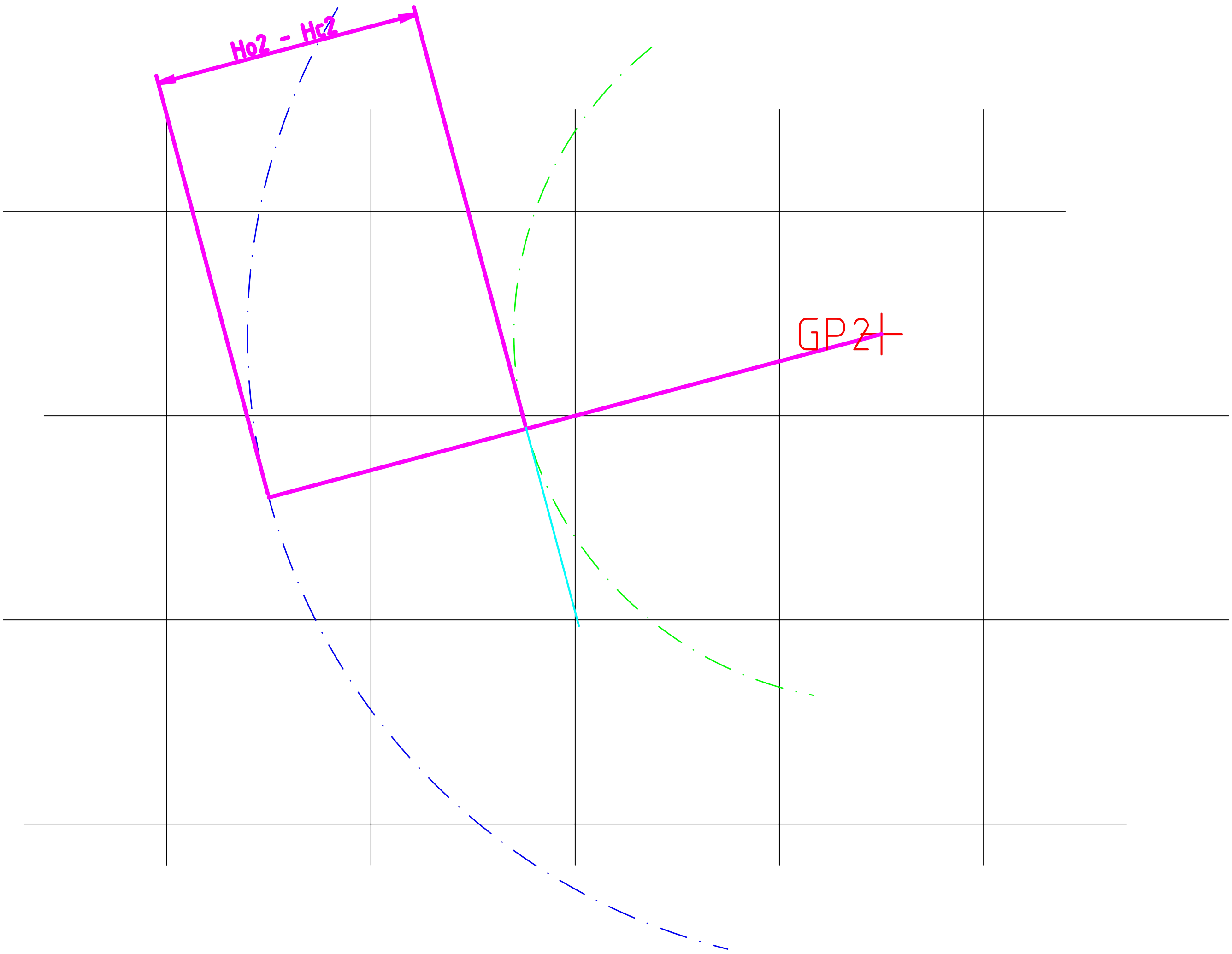
The computed altitude H_C, that is, the angle at which we would observe the object above GP1, defines a circle (blue dashed) around GP1. At all points of this circle the object will be observed with altitude H_C.

Because we are actually at real position RP, we will observe and altitude of H_O, which is different from H_C. This means that we are at a different circle around GP1. Therefore we subtract the difference $H_O - H_C$ from EP towards GP1, that is, via the azimuth line, drawn in magenta. Because we move over a great circle, the difference in arcminutes of $H_O - H_C$ will be the distance we have to correct in Nautical Miles over the azimuth. If $H_O > H_C$ we move towards GP1, and in the other case we move away from GP1. This is because the closer we are to GP1 the larger the altitude angle becomes.
Having done the correction in radius of the circle, we draw a position line which is the tangent to the circle, therefore, it is perpendicular to the azimuth.
This line has been drawn in cyan in the above picture.

Because in normal situations the GP is so far away from our ships position, we cannot draw the GP and the circles in the chart and can only draw the EP, the azimuth and the position line. Of course we don't know the RP, but we know at which radius from the GP it is, and how much that radius differs from the EP radius.

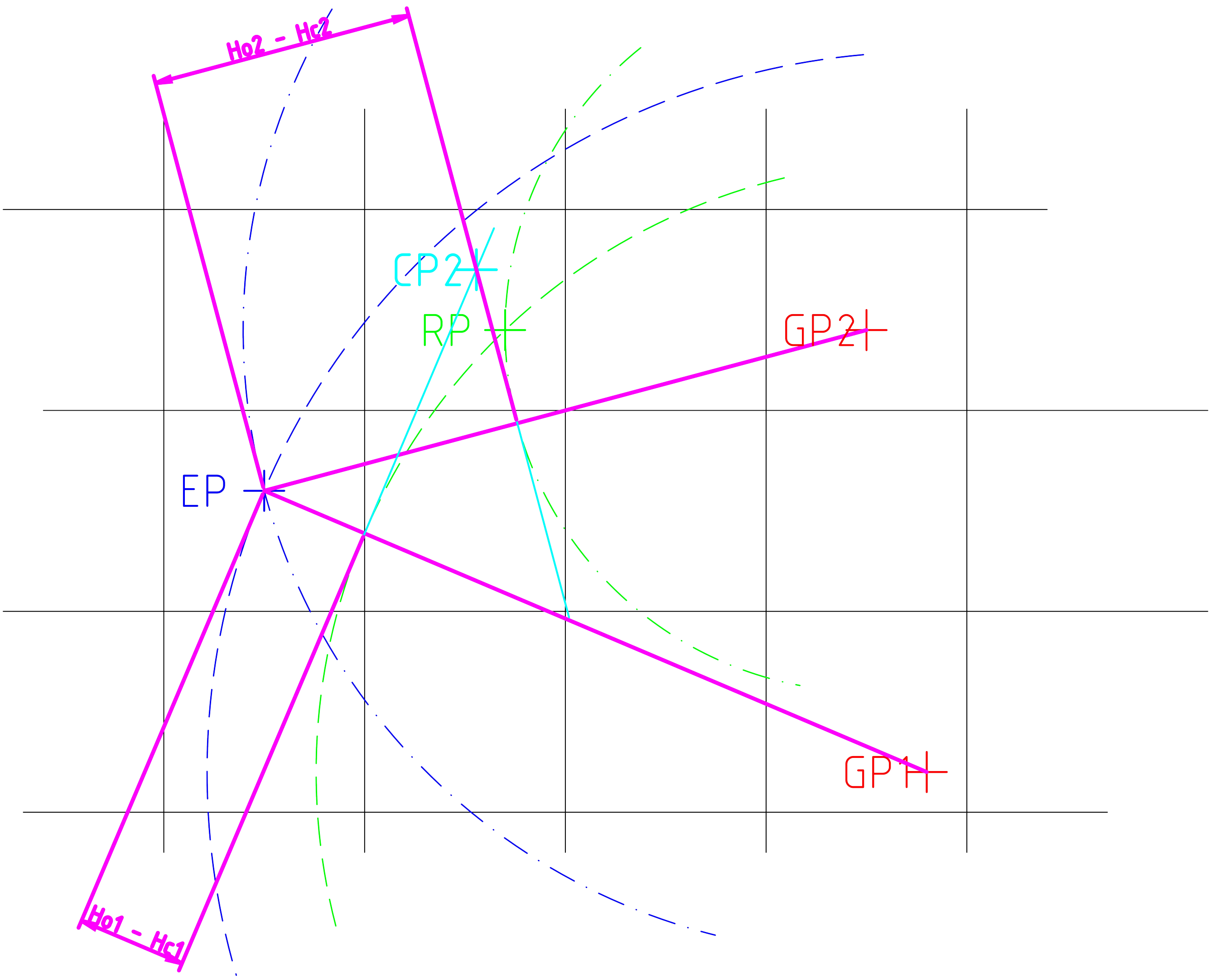
Advantage is: if the GP is so far away, the straight position line is locally a good approximation for the curved line of the circle: the walk-off of the green circular track of the cyan position line will be very small, much smaller than drawn in the picture.

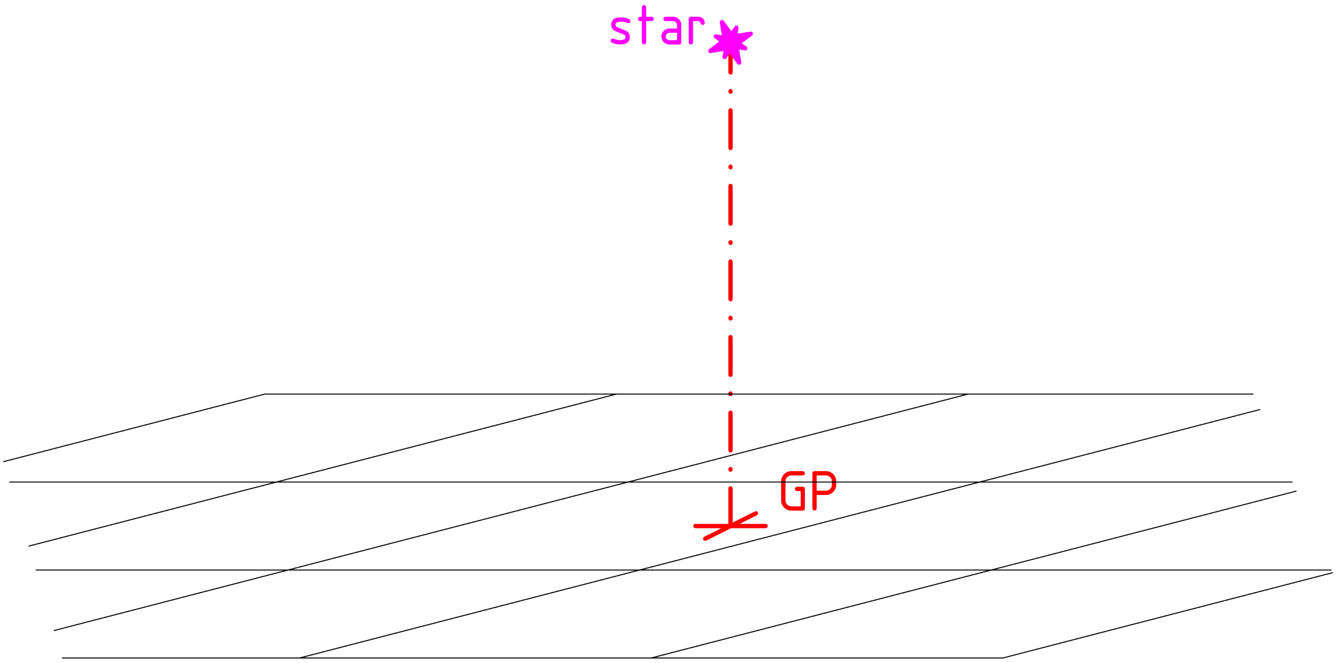
A second and independent observation of object 2 above GP2 gives us a second position line (see picture below).



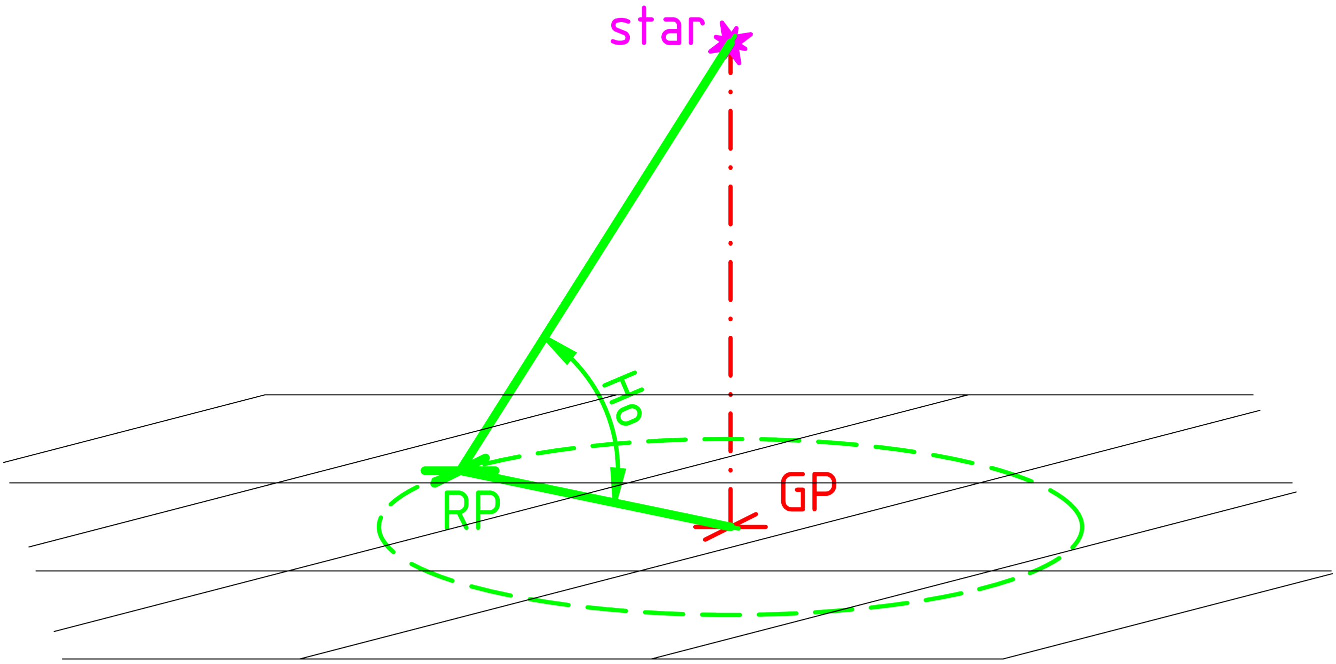
Finally, the point where the two position lines intersect is our constructed ship's position. (See picture below with in cyan CP). Note that in this example the real position (drawn in green) differs pretty much from the constructed position: this is due to the fact that both the GP's are so close to the RP and EP that the position lines are not a very good approximation anymore for the

circles around the GP's.
To guarantee a good approximation the observed altitudes of the objects should not exceed 70°. That means that the circles have radii of over 1200 NM, and a mistake in the EP with respect to the RP of around a degree in latitude/longitude is not a problem: the intersection of both position lines will be closer to the RP than normal measurement errors in measuring the altitude angles with a marine sextant.

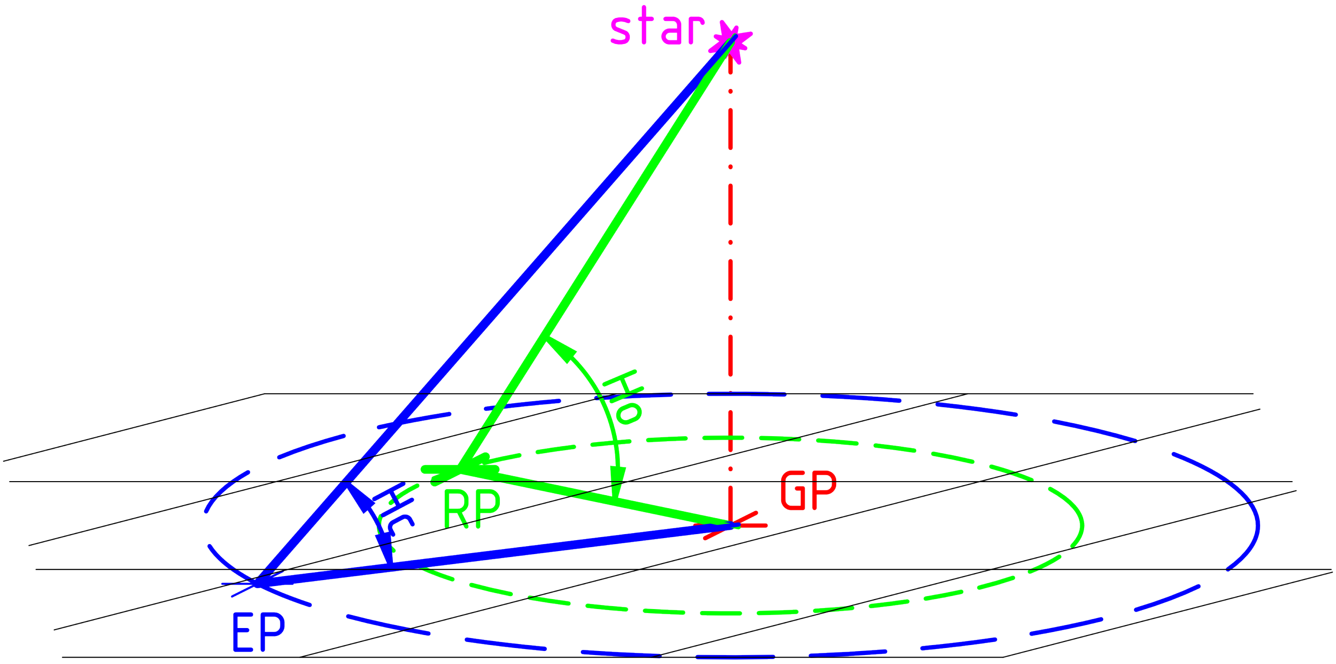




The ground position (GP) of a star in a chart.



The observed altitude (Ho) of the star from the real position (RP). From all points on the green circle around GP the same altitude of Ho will be observed.



The calculated altitude (Hc) for the estimated position (EP). For all points on the blue circle around GP the same altitude of Hc will be computed. The difference between Ho and Hc in arcminutes must be subtracted from the calculated azimuth (blue line between EP and GP) in Nautical Miles to arrive at the green circle of observed altitude.

GP is often very far away and not on the chart. EP, the calculated azimuth, and the difference in altitudes Ho and Hc can be plotted, and with that a position nearby the real position can be constructed.